THE EVOLUTION OF NUMBERS—AN HISTORICAL DRAMA IN TWO ACTS

Author(s): H. E. SLAUGHT

Source: *The Mathematics Teacher*, Vol. 21, No. 6 (OCTOBER, 1928), pp. 305-315
Published by: National Council of Teachers of Mathematics
Stable URL: http://www.jstor.org/stable/27951043
Accessed: 21-06-2016 11:02 UTC

THE MATHEMATICS TEACHER

VOLUME XXI OCTOBER, 1928 NUMBER 6

THE EVOLUTION OF NUMBERS—AN HISTORICAL DRAMA IN TWO ACTS[1]

BY H. E. SLAUGHT

University of Chicago

ACT I: INTERNAL DISSENSION IN THE SECRET SOCIETY OF NUMBERS.

Time: About 1625 A.D. Place: Vestibule of the Secret Chamber of the Society in Paris.

Dramatis Personæ:

The Cardinal: (Representing Whole Numbers, the original Aristocrats.)

The Barbarian: (Representing Negative Numbers, the First Interlopers.)

The Arab: (Representing Zero, the Second Interloper.)

The Hoi Polloi: (Representing Common Fractions.)

The General: (René Descartes in Charge of Personnel.)

Prologue to Act I. (*A young speaker dressed in heraldic costume stands between the parted curtains.*)

My fellow devotees of Mathematics, you will agree that the Number concept is as old as the human race. Let me recall to you some of the struggles through which this concept has come in order to reach its present development. In primitive times there were only a few whole numbers, used for counting, and a few special fractions each with numerator unity. The idea of negative numbers was the outgrowth of the experience of many centuries. It was Descartes in the seventeenth century who conceived the notion of picturing numbers by points on a line extending in both directions from a zero point and thus giving negative numbers and zero a concrete and natural representa-

[1] This playlet may be found helpful in arranging entertainment features for mathematics clubs. It is reprinted here from the AMERICAN MATHEMATICAL MONTHLY by permission of the author and the editor.

tion. The aristocratic whole numbers then became Descartes' positive integers. They had maintained a secret organization, so to speak, for some 3,000 years and had persistently blackballed the negative numbers every time they had sought admission. Likewise zero had been an outcast, a nobody, since the childhood of the race. Even the resourceful Greeks and the mighty Romans were deprived of renown which might have been theirs if they had only been wise enough to admit the humble zero to their clumsy number organizations. What they failed to do the wiser Hindus and Arabs accomplished later on when they brought back to Europe the Hindu-Arabic number symbols and the decimal notation—possibly the greatest invention of all time. Well might Descartes be awarded martial honors for his services as "general in charge of personnel" when he stationed the positive and negative integers and zero as sentinels on his Number Line and then filled in the intervening spaces with the lowly fractions, now recognized as swarming in countless hordes, thus forming the battle front of the mathematical warfare which was to conquer the world.

(*The speaker withdraws from the stage and the curtains open. The Barbarian enters and tries to open the door to the adjoining room, the Secret Chamber of the Society of Numbers. The Cardinal then enters and finds the Barbarian trying to break in. Each actor wears a placard on his breast showing his name in large print.*)

The Cardinal (*Dressed in a cardinal robe*): Is this you, the Barbarian, trying again to gain admittance to the secret chamber of our Number Society?

The Barbarian (*Dressed in a rakish costume*): Yes, General Descartes has told me that we are fully entitled to a place coordinate with you on his new Number Line.

The Cardinal: This is an outrage. We, the Cardinals, have held the major positions in our Society for 3,000 years and it is preposterous now for General Descartes to assign us to positions on the right wing and you to equally important positions on the left.

The Barbarian: But you Cardinals cannot hold the modern battle line alone. It may have been simple enough a few centuries ago when your Number Society was small and there was no need for an extended battle front.

General Descartes (*entering in a costume appropriate to the time. See the Open Court Portraits of Famous Mathematicians*): The Barbarian is quite right. The number army has been only half organized heretofore and many a battle in mathematics has been lost solely because the army had no left wing. What would you think of an army in the military world that should try to do its fighting with one wing while the enemy was free to execute a flank movement on the unprotected side?

The Barbarian: As an example, there was Captain Cardan a hundred years ago, who needed us Negative Numbers the very worst way in order to complete his victory over the Cubic Equation. We were there ready and anxious to do our part but he spurned our assistance and called us "fictitious" and "absurd."

The General: You Cardinals and Barbarians must quit your quarreling and divide the responsibility of sentinel duty on the Number Line, the Cardinals on the right wing and the Barbarians on the left.

The Arab (*Now entering clad in typical Arab costume*): Yes, but that leaves an unguarded sentinel post between the wings and I alone, the long despised and rejected Zero, am the only one qualified to guard this point. I am absolutely the center of things, the *King Pin,* so to speak, in this whole army formation.

The Hoi Polloi (*Clad in midget costume, entering just as the Arab ceases speaking*): But an army must have something more than King Pins and Sentinels! After all, the hordes of Common Fractions are the ones who constitute the real battle front and we are the only ones qualified for this service.

The Cardinal: Well, if we *must* divide the honors with these Barbarians, the Arab and the Hoi Polloi, then our humiliation is complete. We are aristocrats no longer.

The General: But World Progress demands it. You have blocked the world's business for 3,000 years by persistently black-balling these applicants for admission to your Society. Take the Arab, for instance. Without him the ancient Greeks with all their boasted culture could do but little in arithmetic. The mighty Romans could conquer the world in war but they could not master Arithmetic without the help of Zero. It remained for the Hindus and the Arabs to show the world a

decimal number system in which Zero was indeed the "king pin." The Arabs brought this wonderful system to Spain and Leonardo of Pisa introduced it to the rest of Europe; but it took Europe three centuries to fully appreciate this great contribution of the East to the West. Now the Arab and the Barbarian shall come into their own. They shall occupy positions of honor on the Number Line and you Cardinals shall recognize them as your equals. You think it will detract from your glory, but, quite the contrary, it will be your making. You have been sitting around for 3,000 years doing practically nothing but teaching people how to count, while the world's great problems have gone unsolved.

The Cardinal: We recognize our great short-sightedness in this matter and we will now follow your directions.

The General: And another thing! This secrecy business must stop. It began way back in Egypt when Ahmes, the Priest, wrote in the minutes of his famous Papyrus "directions for knowing all dark things." This sort of thing was continued down through the centuries. Someone would solve a problem and carefully conceal the method and then challenge some opponent to find the solution. Even Cardan is said to have gotten his solution of the Cubic from Tartaglia under false pretenses with a promise never to give it away—which promise he deliberately broke.

The Cardinal: Yes, the vow of secrecy was a mistake. We will hereafter do our work in the open.

The General: Finally, you must adopt a Constitution and By-Laws and then examine each applicant for membership on his merits and not act on mere sentiment and prejudice, as you have done all these centuries with the Barbarians and the Arab. The only criterion for membership must be obedience to the By-Laws and allegiance to the Constitution.

The Cardinal: We will do these things, but what about the Arab, the Barbarians, and the Hoi Polloi? Will they also conform?

All in chorus: Yes, we will conform—so say we all of us.

(Two attendants now enter bearing an exhibition of a well-organized Number Line, consisting of a strip of wrapping paper ten or twelve feet long, on which a heavy line is marked off with units and labelled with large figures, positive, negative and zero.

The Cardinal, the Barbarian, Zero, and the Hoi Polloi stand back of this Number Line, while General Descartes stands in front with a pointer indicating the respective subdivisions.)

The General: Henceforth you Cardinals will continue to stand as sentinels on the right wing, and you Barbarians will do duty likewise on the left wing; while the Arab will stand guard at the key position between the two wings, and the Hoi Polloi will fill in the open spaces between the sentinel posts. Thus is the Number Line ready to attack the world's great problems.

(*The Curtain Falls*)

ACT II: INTERNATIONAL ARMISTICE AND PEACE IN THE NUMBER WORLD.

Time: About 1825 A.D. Place: International Open Forum.

Dramatis Personæ:

The Irrational, $\sqrt{2}$: (The Third Interloper.)

The Complex, $\sqrt{-1}$: (The Fourth and Last Interloper.)

The Continuum: (The Solid Front Number Line.)

Professor Dedekind: (Representing the Great Divide.)

Professor Gauss: (Representing the Complex Plane.)

Prologue to Act II. (*Same speaker as in the Prologue to Act I, but now grown old, standing between the parted curtains.*)

My fellow devotees of mathematics, it is now 200 years since General Descartes organized his famous Number Line in which the aristocratic whole numbers were compelled to share the honors of sentinel duty with the negative numbers and zero. During these two centuries a lot of things have happened. When the vow of secrecy was removed from the Number Society as Descartes commanded, there was an immediate renewal of the attempt of the Irrational and the Complex to break into the system. Their struggle for recognition had already been going on for at least 600 years, but encouraged by the success of the Barbarians and the Arab in the seventeenth Century, they then battered harder than ever at the doors of the Number Society till the beginning of the nineteenth century when strong friends like Professors Dedekind and Gauss came to their aid and finally secured their admission. You

are now to hear these actors in the Number Drama speaking for themselves.

(*The herald withdraws and the curtain is opened showing Professors Dedekind and Gauss, in costumes befitting the time* (*see the Open Court Collection of Portraits of Mathematicians*), *walking and talking together.*)

Professor Dedekind: You recall a bold program proposed by General Descartes two hundred years ago when he commanded the Society of Numbers to abolish secrecy and to adopt a Constitution and By-Laws.

Professor Gauss: Yes, I recall the story. It was, indeed, a bold program and it has taken just about two centuries for its accomplishment. Oh, well, that is fast work compared to the slow reforms which General Descartes brought to a climax when he compelled the Cardinals to admit the Barbarians and the Arab to full fellowship in the Society.

Dedekind: It is less than ten years ago since those By-Laws were fully agreed upon and published, namely, the Commutative, Associative and Distributive Laws which really govern our Number System.

Gauss: It took a long time for people to realize that these Laws do actually control the operations of our Algebra and that they are the only criteria for admission to the Society of Numbers.

Dedekind: Of course the Cardinals were the first to adopt these By-Laws, but it took a long time for people to see that the Barbarians, the Hoi Polloi, and Zero all conform to these same Laws, and hence actually belong to the Society just as much as the Cardinals.

Gauss: It does seem awfully slow progress as we look back upon it now, but when we realize how far we have come since Ahmes, the Egyptian Priest, started the organization 3,500 years ago, it seems that we have done wonders in these last two hundred years.

Dedekind: Well, it is up to us to see to it that things move faster in this 19th Century than they did in the preceding centuries. It was a shame and a disgrace to Europe that the Hindu-Arabic symbols and the Decimal notation which were brought to us from Arabia by way of Spain 600 years ago should have

gone almost unnoticed for three centuries before we began to awaken from our long sleep of the Dark Ages to see that Asia had put into our hands what was destined to become the most powerful instrument ever devised by the mind of man.

Gauss: Yes, a great responsibility rests upon us of this generation. Already the members of the Society are getting busy. The Constitution is clear as to our duty. It states that all hands shall add, subtract, multiply, divide, raise to powers and extract roots unceasingly, in season and out of season. That is our job. Thus shall the world's business be transacted.

Dedekind: It is interesting to note that the first and only amendment to the Constitution had to be enacted almost at the very start, namely, that division by zero shall be forever prohibited.

Gauss: Yes, I recall how some well-meaning people have tried to divide by zero and have thought that the quotient was a member of our Society, but have finally been convinced that none of our members can act in that capacity.

Dedekind: There is another curious phenomenon which has caused a lot of trouble in the last two centuries, namely, when our members began to extract roots promiscuously they soon ran across such an anomalous case as the square root of 2 and found that it was neither a Cardinal nor one of the Hoi Polloi. Hence they naturally concluded that such a thing could not belong to our Number Society and so they refused its admission. This case has caused me a lot of trouble and anxiety, but I believe that I now have it satisfactorily explained.

Gauss: That is no more curious than the case that I have been worrying about, namely, the square root of negative one. These two interlopers have caused the Society more trouble than all the orthodox members put together. They have repeatedly applied for admission, and have been consistently rejected for many centuries. Here they come now. Let us hear what they have to say for themselves.

(*The Irrational and the Complex now enter, in costumes appropriate to the age, walking and conversing.*)

The Irrational: I surely have had a rocky time of it. I have tried repeatedly to break into the Number Society but have always failed. They have even told me that I am nobody and have no respectable standing at all. But even Pythagoras

knew better than that 2,300 years ago. He saw that I represent the diagonal of the unit square and that I am just as respectable as the side of the square. Bhaskara in India 700 years ago recognized me as a thoroughly respectable citizen, and the Englishman, Robert Recorde, 300 years ago even wrote a book in which he paid sincere respect to me. But it remained for Professor Dedekind and others of the present time to really vindicate my rights and to atone for the injustice which has been done to me for three thousand years. I want to see him and thank him in person.

The Complex: If you think you have had a rocky time, what do you think of me? I did not have *even one* friend and defendant to do for me what Pythagoras did for you. Not one of the Indian members stood up for me as Bhaskara did for you. I was absolutely an outcast till about 200 years ago when Albert Girard recognized me as a legitimate citizen, while he was solving equations. It is true that Captain Cardan, 100 years earlier, had met me when solving his cubic but he said I was "curious" and "useless." So things went on until a Frenchman by the name of Argand wrote an essay a few years ago in which he defended me and showed why I should be admitted to the Number Society. But nobody paid any serious attention to this essay or to me till Professor Gauss put me on the map and I am hoping to see him and thank him personally for thus atoning for the three thousand years of injustice which I have suffered.

The Irrational (*approaching Professor Dedekind*): Here is Professor Dedekind now. Allow me to thank you, kind sir, for what you have done for me. You cannot know what a relief it is to be admitted to the Number Society after thirty centuries of exile. I know that other friends have been working in my behalf, such as Professors Cantor and Weierstrass, but I am sure your name will go down in history as my emancipator and that the "Dedekind Cut" by which my shackles were broken will forever stand as one of the greatest achievements of mankind. Incidentally you will be pleased to know how gracefully and deferentially the Hoi Polloi have separated themselves into two groups at the "Cut" in order to allow me to take my place on the Number Line. Again, kind sir, let me thank you from the bottom of my heart.

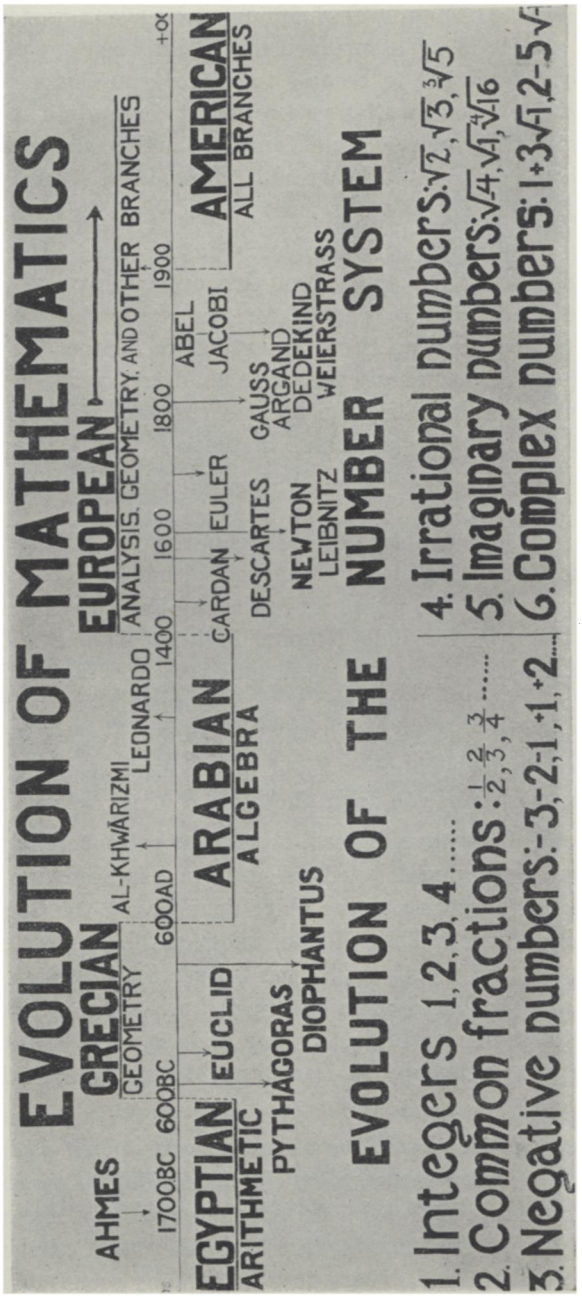
Professor Dedekind: You overwhelm me with your words of praise and gratitude. I assure you that it has been one of the greatest joys of my life to be able to render you this service, especially when I realize that you represent an untold multitude of Irrationals, including the famous π and e, all of whom are now given places on Descartes' Number Line. You have my blessing. Go and take your part in the World's Work.

The Complex (*approaching Professor Gauss*): Ah! Here is Professor Gauss. I am so glad of this opportunity to meet you and to thank you in person for the wonderful way in which you have opened up life's opportunities for me. It was bad enough to be an outcast for three thousand years, but it was the last straw when my friend and companion, the Irrational, was finally given a "place in the sun" on Descartes' Number Line while I was told that there was absolutely no room left there for me.

But just as I was about to die of despair, your brilliant invention, the Gauss Complex Plane, rescued me from oblivion, and now I not only have my place in the sun but I am actually Lord and Master of the whole Number Field, of which the Descartes Number Line is only a small and very special part. I want you to know, kind sir, that I appreciate this great honor and that I am fully conscious of the tremendous responsibility which this wide-open Number Field places upon me. Again, kind sir, let me thank you from the bottom of my heart.

Professor Gauss: You have, indeed, been most patient and long-suffering but your long-delayed recognition has been due not so much to the prejudice of the older members of the Society as to their ignorance. They did not know, and I suppose they could not know, the great possibilities for service that your admission to the Number System will open up. I firmly believe that the 19th Century will justify our highest hopes and will, indeed, see you enthroned as Lord and Master of the whole Number Field.

The Continuum (*A sturdy fellow bearing a placard on which a section of the Number Line is very closely divided to represent the myriads of real numbers*): At last the struggle of mankind to build a Number System adequate to do the world's business is crowned with success. There have been many scenes of con-



This Chart shows the progress of Mathematics, beginning in Egypt, thence to Greece, back to Arabia by way of India, then to Europe by way of Spain, and finally to America—thence to run parallel with Europe for all time.

The dates are given roughly in round numbers. The few names mentioned are chiefly those referred to in the drama.

flict in building this System. There have been jealousies, secret plottings, black-ballings, banishments, but now the mathematical world is at peace. There is complete international understanding and only healthful rivalry. As for myself, I am the symbol of complete union and harmony on the real Number Line. I represent the Solid Front that the Society of Numbers presents to the world. We are now fully prepared to underwrite the world's great problems.

(*The drama closes with all the members of the Company on the stage beneath a banner*[2] *on which is a graphic representation of the Real and Complex numbers. General Descartes and Professors Dedekind and Gauss are standing in front of the Number Group, and all join in singing a stanza of "Hail, hail, the gang's all here."*)

[2] The accompanying half-tone may be helpful in visualizing the historical setting. The original size was seven by three feet. Any club giving the play would do well to make a preliminary study of the historical references. The dates here given are, of course, only roughly approximate in round numbers.